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LETTERS

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An approach to wall modeling in large-eddy simulations

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Channel flow with friction Reynolds number Re_τ as high as 80 000 is treated by large-eddy simulation at a moderate cost, using the subgrid-scale model designed for detached-eddy simulations. It includes wall modeling, and was not adjusted for this flow. The grid count scales with the logarithm of the Reynolds number. Three independent codes are in fair agreement with each other. Reynolds-number variations and grid refinement cause trades between viscous, modeled, and resolved shear stresses. The skin-friction coefficient is too low, on the order of 15%. The velocity profiles contain a ''modeled'' logarithmic layer near the wall and some suggest a ''resolved'' logarithmic layer farther up, but the two layers have a mismatch of several units in U^+. © *2000 American Institute of Physics.* [S1070-6631(00)01607-X]

The principal challenge in the large-eddy simulation (LES) of wall-bounded flows is the rate at which computing cost increases with Reynolds number. If the buffer layer and its streaks are resolved, the subgrid-scale stresses are hardly larger than the viscous stresses, the cost scaling is identical to direct numerical simulation (DNS), and the technique comes close enough to be called quasi-DNS, or QDNS.[1] If the streaks and some of the eddies that populate the logarithmic layer are not resolved, we have ''wall modeling'' and the cost scaling can be considerably more favorable. However, it seems to us unavoidable that much more empiricism will be involved than in QDNS, an empiricism which implies the law of the wall.

Detached-eddy simulation (DES) was conceived for external aerodynamics, with the juxtaposition of thin boundary layers and massive-separation regions,[1] and has given encouraging results for such flows.[2] In its intended use, the boundary layers are treated entirely with Reynolds-Averaged Navier–Stokes (RANS) equations. The user directs the model to operate in that mode by creating a ''RANS grid'' with a large spacing parallel to the wall, compared with the boundary-layer thickness: $\Delta_\parallel \gg \delta$. In the separated regions, good accuracy is expected once the grid spacing in all directions is far smaller than the size of the region: $\Delta \ll L$.

The present application of DES, in a ''LES grid'' with $\Delta_\parallel \ll \delta$, is unnatural but has two motives. The first is to predict the behavior of DES in the recovery of a thickened boundary layer, such that δ comes to exceed $\Delta_\parallel$. The second is the fact that the formulas of DES provide a viable subgrid-scale (SGS) model on a LES grid, with built-in wall modeling. Its only adjustable constant to date was set in homogeneous turbulence.[2] We could not predict whether wall-bounded turbulence would be sustained, or how accurate the solution would be, all of this depending on the grid. We have ''LES'' in the title to reflect how, on the present grids, DES amounts to LES with a plausible one-equation SGS model and wall modeling. In fact we have a *suboptimal* SGS model, because the parent model is constrained by its calibration in RANS mode.[3]

A tangential motive is to stimulate discussion on the ambitions and limitations of wall modeling, in terms of grid spacing and of fidelity of the resolved field. Many proposals for near-wall SGS improvements do not depart from QDNS. We believe that a minimal ambition for wall modeling should be to remove limits on the wall-parallel grid spacing in wall units: $\Delta_\parallel^+$ could be large, provided $\Delta_\parallel/\delta$ is small. That goal is well accepted in some circles, but the normal

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TABLE I. Simulation parameters.

Run	Re_τ	$N_x\times N_y\times N_z$	$\Lambda_x\times\Lambda_z$	$\Delta_\parallel$	Δy_{cl}	$\Delta_\parallel^+$	Δy_w^+	ΔC_f
A1	180	128×77×41	$4\pi\times 4/3\pi$	0.1	0.035	18	0.5	−1%
A2	2000	64×64×32	$2\pi\times\pi$	0.1	0.09	200	0.8	−19%
A3	20000	64×90×32	$2\pi\times\pi$	0.1	0.1	2000	0.7	−14%
A4	20000	128×120×64	$2\pi\times\pi$	0.05	0.05	1000	0.2	−13%
A5	80000	64×128×32	$2\pi\times\pi$	0.1	0.07	8000	0.4	−14%
B1	3000	64×64×32	$2\pi\times\pi$	0.1	0.12	295	0.3	−22%
C1	2750	64×64×32	$2\pi\times\pi$	0.1	0.11	270	1.1	−11%
C2	23200	64×128×32	$2\pi\times\pi$	0.1	0.07	2280	1	−5%

grid spacing $\Delta_\perp$ calls for further decisions. In DES as it stands, it is strictly limited in wall units: $\Delta_\perp^+=O(1)$ at the wall; the grid cells are shallow. In contrast, some wall-modeling proposals aim at much larger values such as $\Delta_\perp^+\approx 50$ or even unlimited values (while maintaining $\Delta_\perp \ll \delta$, of course); the grid cells are not shallow. This echoes the ''wall functions'' of RANS modeling. In good modeling proposals, clear statements regarding $\Delta_\parallel$ and $\Delta_\perp$ allow the calculation of the cost scaling with Reynolds number, which is highly relevant to planning.[1] Regarding the fidelity of the fields, we believe that any type of wall modeling will produce unrealistic coherent structures in a kind of ''super-buffer layer'' at the bottom of the LES region.

The present DES model is a simple derivative of the Spalart-Allmaras (S-A) one-equation eddy-viscosity RANS model.[3] The DES modification concerns the destruction term, and hinges on the length scales d and $\tilde{d}$. In S-A, d is the distance to the nearest wall and expresses the (inviscid) confinement of the eddies by that wall. In the DES model, we replace d with $\tilde{d}$, which is defined as

$$\tilde{d}\equiv\min(d,C_{DES}\Delta)\quad \text{with}\quad \Delta\equiv\max(\Delta x,\Delta y,\Delta z).$$

The role of Δ is to allow the energy cascade down to the grid size; roughly, it makes the pseudo-Kolmogorov length scale, based on the eddy viscosity, proportional to the grid spacing. We use the largest dimension of the grid cell, in contrast with the often-used cube-root definition of Δ.[1]

The model is as follows. The transition terms were removed from the S-A model[3] and would have no impact except maybe near the QDNS regime. The SGS stresses are given by $-\overline{u_i'u_j'}=\nu_t(\partial u_i/\partial x_j+\partial u_j/\partial x_i)$ where u_i is the resolved field. The eddy viscosity ν_t is given by

$$\nu_t=\tilde{\nu}f_{v1},\quad f_{v1}=\frac{\chi^3}{\chi^3+c_{v1}^3},\quad \chi\equiv\frac{\tilde{\nu}}{\nu}.$$

ν is the molecular viscosity. $\tilde{\nu}$ is the working variable and obeys the transport equation

$$\frac{D\tilde{\nu}}{Dt}=c_{b1}\tilde{S}\tilde{\nu}+\frac{1}{\sigma}[\nabla\cdot((\nu+\tilde{\nu})\nabla\tilde{\nu})+c_{b2}(\nabla\tilde{\nu})^2] -c_{w1}f_w\left[\frac{\tilde{\nu}}{\tilde{d}}\right]^2.$$

Here S is the magnitude of the vorticity,

$$\tilde{S}\equiv S+\frac{\tilde{\nu}}{\kappa^2\tilde{d}^2}f_{v2},\quad f_{v2}=1-\frac{\chi}{1+\chi f_{v1}}.$$

The function f_w is

$$f_w=g\left[\frac{1+c_{w3}^6}{g^6+c_{w3}^6}\right]^{1/6},\quad g=r+c_{w2}(r^6-r),\quad r\equiv\frac{\tilde{\nu}}{\tilde{S}\kappa^2\tilde{d}^2}.$$

The wall boundary condition is $\tilde{\nu}=0$. The constants are $c_{b1}=0.1355$, $\sigma=2/3$, $c_{b2}=0.622$, $\kappa=0.41$, $c_{w1}=c_{b1}/\kappa^2+(1+c_{b2})/\sigma$, $c_{w2}=0.3$, $c_{w3}=2$, $c_{v1}=7.1$.

In a structured grid, Δx and Δz are independent of y while Δy is refined near the wall, so that there is a layer near the wall in which $\tilde{d}=d$, loosely called the ''RANS region,'' and a region away from the wall in which $\tilde{d}=C_{DES}\Delta$, called the ''LES region.'' Tests in isotropic turbulence give us confidence in a (well-resolved) LES region, a good value for C_{DES} being 0.65.[2] In the RANS region, we are confident that the S-A calibration of the wall and viscous terms will serve well, and produce a ''buffeted logarithmic layer,'' provided the grid patch of size $\Delta x\times\Delta z$ is large enough to contain a large sample of streaks. A first estimate is that $\Delta_\parallel$ should be at least several hundred, since a typical streak spacing is 100. Between the RANS and LES regions lies a ''gray region,'' to which we can only resort for testing.

The grid scaling with Reynolds number, in DES, is as follows. For a given level of accuracy only the normal grid depends on Reynolds number, and only in the near-wall region (in the core region of the channel we have $\Delta_\perp\approx\Delta_\parallel$). We maintain $\Delta_\perp^+\approx 1$ for the first point, and a stretching ratio of about 1.15 between grid cells in the log layer. If that ratio is applied systematically, a factor of 10 in Re_τ adds only 17 grid layers for each wall: the evolution is logarithmic. Some of our grids had somewhat lower ratios and/or lower $\Delta_\perp^+$, which raised the cost over the estimate. The scaling of the time step is also very mild. As a result, the computing cost for a given accuracy increases more slowly than powers of Re_τ, and the economic incentive to use ''wall functions'' to relax the $\Delta_\perp^+\approx 1$ requirement is weak.

In this communication, results are presented from three computing teams with independent numerics, similar grid designs, and tangible differences in the y distribution. The range of Reynolds number Re_τ based on friction velocity u_τ and channel half-depth h is from 180 (a QDNS) to 80 000 (a full LES), but the number of grid points is of the same order, and does not exceed 1 million. The simulations were conducted on personal computers and work stations; this is not a

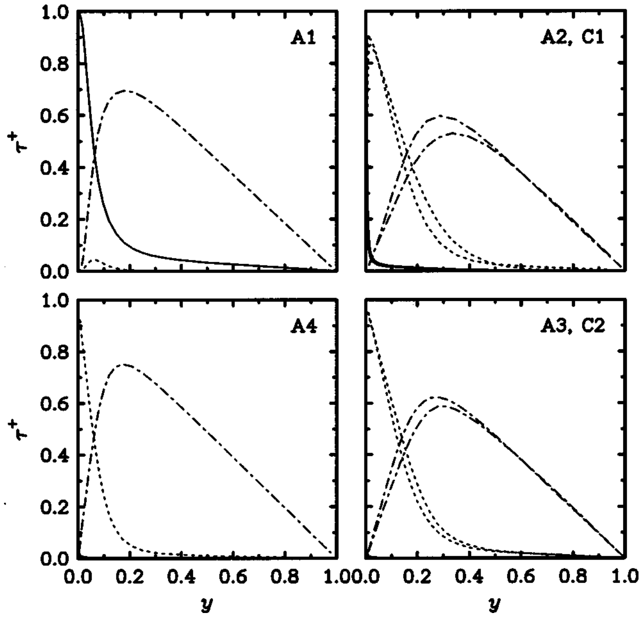

FIG. 1. Shear stresses: viscous (—); modeled (---); resolved (-–-).

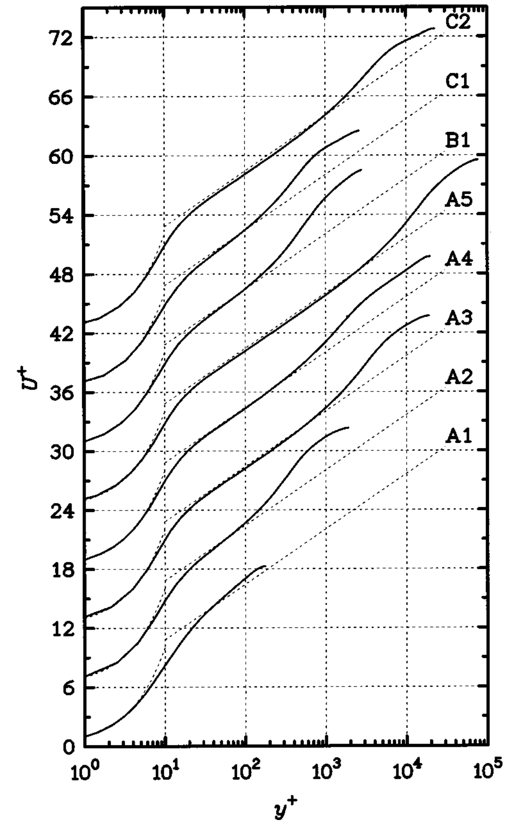

FIG. 2. Velocity: DES (—); $U^+=y^+$ and $U^+=\log(y^+)/0.41+5.2$ (---).

computing ''grand challenge.'' All three codes use finite-difference discretizations. The schemes are centered (therefore dissipation free) and second-order accurate for the momentum equation. Some codes use upwind differences for the transport equation of the SGS model. The time differencing is by second-order mixed explicit–implicit schemes. This technology is well understood, and each code was tested on channel DNS.

The major parameters and error measure, the error in skin-friction coefficient ΔC_f, are in Table I. The letters A, B, C identify the codes. The lengths not in wall units are normalized by h. The domain sizes are large enough not to seriously disturb the accuracy. The grids are fairly coarse, compared with an earlier estimate that requires 20 points per boundary-layer thickness ($\Delta_\parallel/h=0.05$).[1] All runs but A4 have $\Delta_\parallel/h=0.1$ instead. If $\Delta_\parallel/h$ is raised from 0.1 to 0.2, the resolved turbulence usually ''dies,'' leading to a steady one-dimensional solution. This solution is inaccurate, because $\tilde{d}<d$, so that the model does *not* reduce to the RANS model; at $\mathrm{Re}_\tau=2000$ with $\Delta_\parallel/h=0.2$, the C_f is too low by 47%. Full RANS is recovered only when $\Delta_\parallel/h\geqslant1.5$. Identifying this ''danger zone'' is consistent with the first motive cited previously.

All cases have $\Delta x=\Delta z$, which is the *a priori* policy of DES. The natural tendency, coming from QDNS, is to set $\Delta x\approx3\Delta z$. This assumes that the smallest resolved flow structures will resemble streaks, which is not obvious. More important, a 3-to-1 grid cell is undesirable in practice since it requires a previous knowledge of the wall-shear direction.

Figure 1 shows the viscous, modeled, and resolved shear stresses for six cases. The viscous stress $\overline{\nu u_y}$ is appreciable at $\mathrm{Re}_\tau=180$ (A1), much smaller at $\mathrm{Re}_\tau\approx2500$ (A2, A3, C1), and cannot be seen on a linear axis at $\mathrm{Re}_\tau\geqslant20\,000$ (A4, C2). Of the two turbulent stresses, the modeled stress $\overline{\nu_t(u_y+v_x)}$ dominates near the wall on the coarser grids, and the resolved stress $-\overline{uv}$ takes over near the center of the channel. Case A1 has very little modeled stress, a mark of a QDNS (the grid can support a DNS, although not a very accurate one). Comparing cases A2 and C1 or A3 and C2 shows the modest effect of numerics and y grids, although far from negligible relative to the C_f error. Comparing A3 and A4 shows how grid refinement, by a factor of 2, greatly suppresses the modeled stress and expands the resolved stress. Comparing Cases A2 with A3, or C1 with C2, pairs which have the same $\Delta_\parallel$ but different Re_τ, shows little change for the resolved stress. The trade is only between viscous and modeled stresses. In summary, the stresses have the expected behavior, and suggest good numerical stability of DES.

Figure 2 shows the mean velocity profiles. Case A1 compares well with DNS.[4] When estimating the error in the skin-friction coefficient C_f at other Reynolds numbers, we extrapolated from the Moser–Kim–Mansour value at $\mathrm{Re}_\tau=590$, which had the same domain size,[4] assuming that the defect law applied and $\kappa=0.41$. Specifically, we took $U_{\mathrm{cl}}^+=21.26+\log(\mathrm{Re}_\tau/587)/0.41$ as the exact value for the centerline velocity (and $C_f\equiv2/U_{\mathrm{cl}}^{+2}$). Wei and Willmarth's high-Reynolds-number case[5] would put U_{cl}^+ about 0.25 higher (still assuming $\kappa=0.41$) and therefore slightly improve the agreement. The profiles collapse up to y^+ about 20 and blend into a logarithmic law. The solution is a quasi-steady RANS, and most of the shear stress is carried by the model; this is the ''modeled log region.'' Most profiles are slightly below the S-A log law ($C=5.2$). This is not due to DES. It is due to the coarse grids, and does not happen in Case A4. After that, the profile rises above the log law significantly more than the DNS indicates, resulting in low C_f values. The fine-grid case A4 brings little improvement in the skin friction, relative to A3. This is normal, within a range; the height at which the shear stress switches from modeled to resolved does not alter the dynamics of the super-buffer layer, which set the relative levels of the wall layer and defect layer. The steep region in the super-buffer layer near $y=1.5\Delta_{\parallel}$ echoes the true buffer layer, near $y^{+}=10$.

The C1/C2 pair, which most closely follows the DES gridding policy, comes close to obeying the defect law in the sense that the upper parts of the profiles are almost an exact translation of each other (to be rigorous, defect-law behavior could be an artifact of the gridding policy, since it rests on the same assumptions). It also suggests a ''resolved log region'' ($0.7\times10^4<y^{+}<1.7\times10^4$ for C2), but both the effective Karman constant κ and the intercept C (in $U^{+}=\log(y^{+})/\kappa+C$) are much too high: $\kappa\approx0.55$ and $C\approx12.8$. Case A4 with its fine grid gives a much better value, $\kappa\approx0.425$, but of course C is high, near 8.6. These two mismatches have different interpretations. A mismatch in C is not unexpected since the details of the ''DES buffer layer'' or gray region between the modeled-stress region and the resolved-stress region cannot be predicted in advance. In other words, until the modeling is adjusted there is no reason why C should be very accurate in the resolved log region. On the other hand, an inaccurate κ in the resolved-stress region can only be attributed to insufficient resolution. With the grid spacing roughly equal to $h/10$ and second-order differencing, this is not a surprise.

In summary, the DES model was applied without adjustment as a SGS model in the LES of channel flow. The simulations sustain turbulence, and the results are stable and fairly accurate. This raises hope that gradual improvements could lead to a simple, stable, and accurate approach to wall modeling. Having three codes gives us a bracket for the discretization effects, as opposed to modeling effects. It was found that the model as it stands leads to an upper part of the velocity profile that is tangibly too high. We made simple attempts, such as rounding the distribution of $\tilde{d}$, compared with the simple ''minimum'' formula above. These tended to worsen the disagreement (varying C_{DES} does not help either). Given time, improvements may come along two branches: one which preserves the S-A calibration so as to remain useable in DES, and another aimed purely at LES. A crude measure for the latter purpose would be to reset c_{v1} to 4, so as to lower the modeled log layer.

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