Extracting the Acoustic pressure field from Large Eddy Simulation of confined reactive flows

Camilo F. Silva*
CERFACS, 31057 Toulouse, France
Franck Nicoud †
Université Montpellier II, 34095 Montpellier, France
Stéphane Moreau ‡
Université Sherbrooke, 2500, boul. de l'Université Sherbrooke (Québec), Canada J1K 2R1

Today, much of the current effort in the field of combustion noise is the development of efficient numerical tools to calculate the noise radiated by flames. Although unsteady CFD methods such as LES or DNS can directly provide the acoustic field radiated by noise sources, this evaluation is limited to small domains due to high computational costs. Hybrid methods have been developed to overcome this limitation. In these schemes, the noise sources are decoupled from the radiated sound. The sources are still calculated by DNS or LES codes whereas the radiated sound is evaluated by acoustic codes.

These two approaches (direct and hybrid) have been widely used to assess combustion noise radiated by open flames. On the contrary, not significative work has been done for evaluating noise radiated by confined flames. Moreover, several problems can arise when attempting to compare direct to hybrid methods results from confined combustors. While almost a pure acoustic pressure field is obtained from direct computations when considering an open flame, for confined flames this statement does not hold: hydrodynamic pressure fluctuations play an important role. In the present paper the assessment of combustion noise is conducted by both direct (LES) and hybrid computations in a premixed swirled combustor. A method to extract the acoustic field from the entire pressure field given by LES is proposed.

keywords: combustion noise, acoustic analogy, direct computations, hybrid computations.

I. Introduction

Large Eddy Simulation (LES) has become an important tool for the simulation and analysis of turbulent flows. It offers the best promise in the foreseeable future for the estimation of noise from flows at Reynolds Numbers of interest in both open and closed systems. In aeroacoustics, LES plays an important role in the study of aerodynamical generated noise of numerous practical cases that range from air jets, high-lift devices or landing gears in an aircraft to the rear-view mirror of a car or the fan of a wind mill.[1,2] Combustion noise is, on the contrary, less understood than aeroacoustics. This is due to the different physical phenomena implied such as the addition of unsteady heat release to the already turbulent flow. Still LES has been successfully applied to partially premixed and non-premixed open flames[3,4] as well as in more complex cases such as gas turbine combustors.

Computational techniques for the estimation of sound can be classified into two broad categories: direct computations and indirect, or hybrid, computations. LES is well presented in these two categories. Direct computations resolve the flow field together with the sound radiation. A compressible LES code is therefore

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*Corresponding author. Ph.D. Student, CFD Team, 42, Av. Gaspard Coriolis; silva@cerfacs.fr. Tel: (33).(0)5.61.19.31.10

†Professor, Université Montpellier II, I3M - CNRS UMR 5149 - CC51, franck.nicoud@univ-montp2.fr

‡Professor, GAUS, Mechanical Engineering Department, Université de Sherbrooke, stephane.moreau@usherbrooke.ca.

required in addition to high-resolution numerical schemes in order to minimize both dispersion and dissipation. Moreover, the computational domain must be large enough to include the sources of noise as well as part of the acoustic near field.[6] Very expensive computational costs can arise since hydrodynamic and acoustic scales differ to a large amount in typical applications where the Mach number is moderate. This is even more true when dealing with thermoacoustics since the transport equation of each species must be considered in order to solve the problem of compressible multicomponent reactive flows.

In hybrid approaches, the computation of sound is made at two different levels: 1) model the sources of noise which requires a proper estimation of the flow and the flame dynamic properties and 2) predict the far field acoustic radiation due to the different noise sources. Acoustic propagation is calculated based on equations relevant to acoustic phenomena. The derivation of a wave equation governing sound propagation in an arbitrary mean flow (and therefore accounting for mean flow-acoustic interactions) remains a difficult and controversial task in aeroacoustics.[7]

These theoretical formulations are satisfactory for open systems, i.e. when the acoustic fluctuations produced by the source propagate to the infinite and anechoic far-field. Moreover, in these cases, it is relatively easy to distinguish pure acoustics from hydrodynamic pressure fluctuations in the region of interest (farfield): hydrodynamic pressure fluctuations are negligible in the far field since they typically decay at least as the inverse third power of the distance to the sources.[15] Less is known about aeroacoustics in confined domains where acoustic and hydrodynamic pressure fluctuations are both present. Interesting developments have been done to account for turbulence-body interaction[16,17] . More recently, Schram used a modified Curle's analogy combined with a boundary element method (BEM) for evaluating the acoustic field produced by a non-compact turbulent source in a confined domain.[18]

In the field of combustion noise, it seems that no significative work has been done for evaluating the noise produced by confined flames. In confined reactive flows, interactions exist between the flame, the turbulent flow and the walls of the system. Moreover, hydrodynamic pressure fluctuations might be important and should not be neglected. Consequently, the task of evaluating acoustics from a complete signal of fluctuating pressure becomes difficult. It has been shown in[19] that pressure waves coming from LES are highly disturbed by turbulence and hence a proper comparison between LES and hybrid simulations sometimes is unachievable. The general objective of this study is to investigate some methods to decouple hydrodynamics from acoustics in pressure signals resulting from Large Eddy Simulations of reactive confined flows.

II. Combustion noise through Phillips' analogy

The first attempt to include inhomogeneities of the mean flow into the acoustic wave operator is due to Phillips[20] who derived the following expression:

$$\begin{aligned}\frac{d^2\pi}{dt^2} - \frac{\partial}{\partial x_i}\left(c^2 \frac{\partial \pi}{\partial x_i}\right) &= \frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i} + \frac{d}{dt}\left(\frac{\gamma - 1}{\rho c^2}\dot{q}\right) \\ &+ \frac{d}{dt}\left[\frac{\gamma - 1}{\rho c^2}\left(\nabla\cdot(\lambda\nabla T) - \rho\sum_k Y_k c_{p,k}\mathbf{v}_k\cdot\nabla T + \tau : \nabla\mathbf{u}\right)\right] \\ &- \frac{\partial}{\partial x_i}\left(\frac{1}{\rho}\frac{\partial \tau_{ij}}{\partial x_j}\right) + \frac{d^2}{dt^2}(\ln r)\end{aligned} \tag{1}$$

where π is function of the logarithm of the pressure $\pi = (1/\gamma)\ln(p/p_\infty)$. The first term on the RHS is related to the noise created by turbulence. The second term is the monopole source of noise due to the unsteady heat release induced by the flame. The third one is linked to the noise produced by molecular transport whereas the gradient of the viscous tensor appears in the fourth term. Finally, the last term is known as the non-isomolar combustion source of noise.

In order to simplify this equation, one may consider different realistic assumptions in order to evaluate the acoustics for low Mach number reactive systems.[21] Therefore, it is stated that:

- The pressure level of the oscillations are small compared to the local mean pressure. $p'/p_0 \ll 1$.
- The system is nearly isobaric so that p_0 is nearly constant.
- The mean flow is small so that the convective terms in the equation are negligible.

As a consequence, the acoustic wave equation for low Mach number reacting flows reads

$$\nabla \cdot (c_0^2 \nabla p') - \frac{\partial^2 p'}{\partial t^2} = -(\gamma - 1)\frac{\partial \dot{q}'}{\partial t} - \gamma p_0 \nabla \boldsymbol{v} : \nabla \boldsymbol{v} + \frac{\gamma p_0}{W_0}\frac{\partial^2 W'}{\partial t^2} \tag{2}$$

where c_0, p, γ, $\dot{q}$, $\boldsymbol{v}$, W represent respectivelly the speed of sound, the pressure, heat capacity ratio, the heat release rate, the velocity vector and the mixture molar weight. The symbols $()_0$ and $()'$ define respectivelly mean quantities and fluctuation quantities. As it can be noticed in the left hand side of eq. 2, the speed of sound c is placed inside the divergence operator. This ensures to capture acoustic fluctuations with strong variation of the mean temperature as it occurs close to the flame front.

In the combustion case exposed in this paper, the non-isomolar combustion noise does not play an important role since the reactant mixtures are highly dilluted in nitrogen. Further on, the aerodynamic source of noise is considered small with respect to the noise source associated with the perturbation of the heat release rate.[22] The inhomogeneous wave equation then reduces to

$$\boxed{\nabla \cdot (c_0^2 \nabla p') - \frac{\partial^2 p'}{\partial t^2} = -(\gamma - 1)\frac{\partial \dot{q}'}{\partial t}} \tag{3}$$

Under the harmonic oscillation assumption, in which $p'(x,t) = \hat{p}(x)e^{-i\omega t}$, Eq. 3 becomes

$$\begin{cases} \nabla \cdot c_0^2 \nabla \hat{p} + \omega^2 \hat{p} = -i\omega(\gamma - 1)\hat{\dot{q}} & \text{in } \Omega \\ + \text{ Boundary Conditions} & \text{on } \Gamma \end{cases} \tag{4}$$

where $\omega = 2\pi f$. The quantities $\hat{p}$ and $\hat{\dot{q}}$ are complex amplitudes which depend on space only.

III. Experimental Configuration

This article describes the evaluation procedure of noise due to the combustion within a swirled premixed combustor[26, 27] (EC2 Combustor) performing both direct and indirect computations. The experimental study is carried out in the laboratory EM2C (École Centrale Paris). The EC2 combustor consists in two geometrical identical stages for air-fuel injection, a premixer and a combustion chamber. The flame is controlled by the Fuel-Air ratio imposed in each of the two stages and stabilized by a swirled premixed. The test rig accounts for 7 different measurement points of pressure (denoted $M1$ to $M7$ in fig.1) placed at equivalent distances along the combustor.

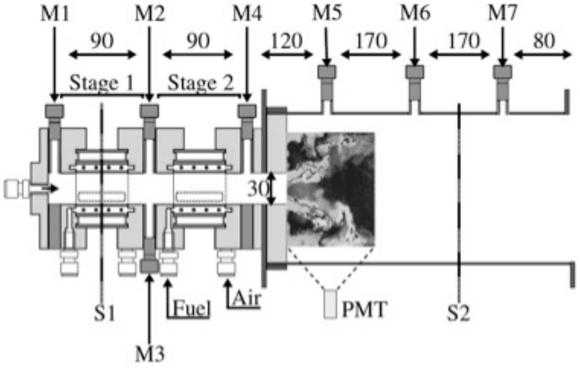


Figure 1. Two staged swirled premixed combustor. (Courtesy of École Centrale Paris)

IV. Combustion noise Analysis

AVBP, developed by CERFACS, is the parallel solver used for the LES computations.[28] In this study, the full compressible Navier Stokes equations are solved on hybrid (structured and unstructured) grids with second order spatial and temporal accuracy. Subgrid stresses are described by the Smagorinksy model. The flame/turbulence interactions are modeled by the Thickened Flame (TF) model.[29] The spatial discretization is based on the finite volume method with a cell-vertex approach, combined to a numerical scheme derived from the Lax-Wendroff scheme. AVBP has been validated/used for a considerable number of configurations.[30–32]

In[19] the computation of the combustion noise from the EC2 combustor was carried out with two different grid resolutions: A 'coarse' mesh of 3 million cells and a 'refined' mesh of 10 million cells. It was found that both meshes reproduced sufficiently well PIV measurements[33] when considering mean quantities. On the other hand, when RMS values were considered only the refined mesh results agreed well with the experimental results. Figure 3 shows the axial velocity profiles along the cuts displayed in Fig. 2

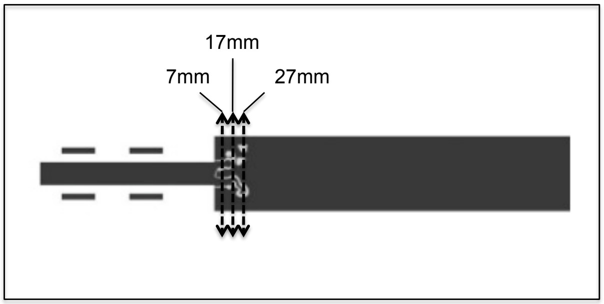
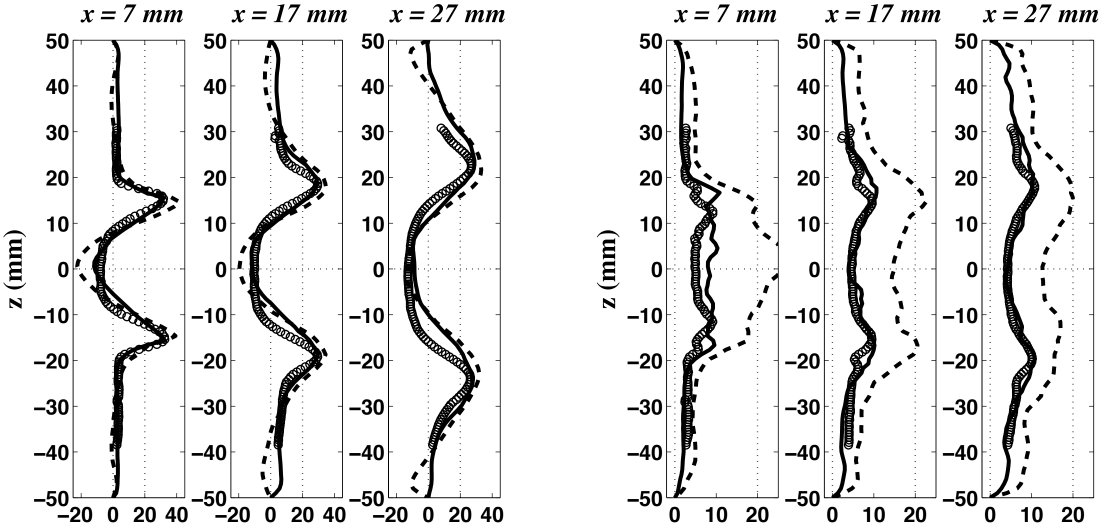


Figure 2. Section cuts for velocity profiles



(a) Mean Axial Velocity

(b) RMS Axial Velocity

Figure 3. Velocity Profiles: ○ Experimental PIV measurements

− − − LES 3 million cells, ——— LES 10 million cells

Reproducing flow dynamics in a proper way (see RMS values for the refined mesh in Fig. 3) does not mean necessarily a good estimation of combustion noise. Computation of combustion noise is very challenging, since a good prediction of the rate of change in the flame surface area must be achieved.[34] Figure 4 shows the Sound Pressure Level (SPL) spectrum at microphone 7 (see the location of $M7$ in Fig. 1) for the computations with the refined and coarse meshes as well as the experimental measurements.[33]

It has been observed that in order to correctly evaluate the dynamics of a flame and the acoustics generated by this one, the resolution of the computation (resolution of the mesh and order of the numerical scheme) is of significative importance. The gap seen in fig. 4 between experimental results of the acoustic spectrum and the refined LES is problably due to the lack of resolution of the computation. For this purpose,

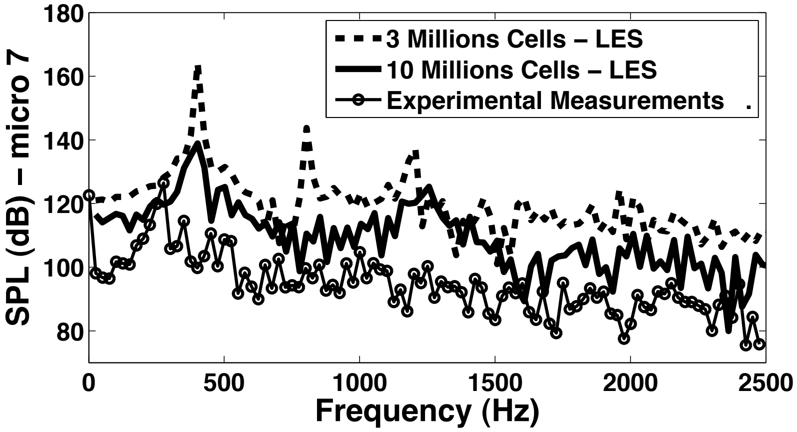


Figure 4. Sound Pressure Level

a computation with 50 million cells is currently being conducted.

Although the estimation of combustion noise through direct computation does not match experimental measurements, it is still a good exercise of analysis to compare direct computation results to those given by hybrid methods. For this comparison to take place, it is assumed that pressure fluctuations from LES contain a negligible amount of hydrodynamics. This comparison is carried out for the 10 million cells mesh and is independent from experimental data.

Overall good agreement is found between both direct and hybrid approaches, as shown if Figs. 5 and 6 which shows the sound pressure levels obtained for microphones 5, 6 and 7 (see the location of $M5$, $M6$ and $M7$ in fig. 1) .

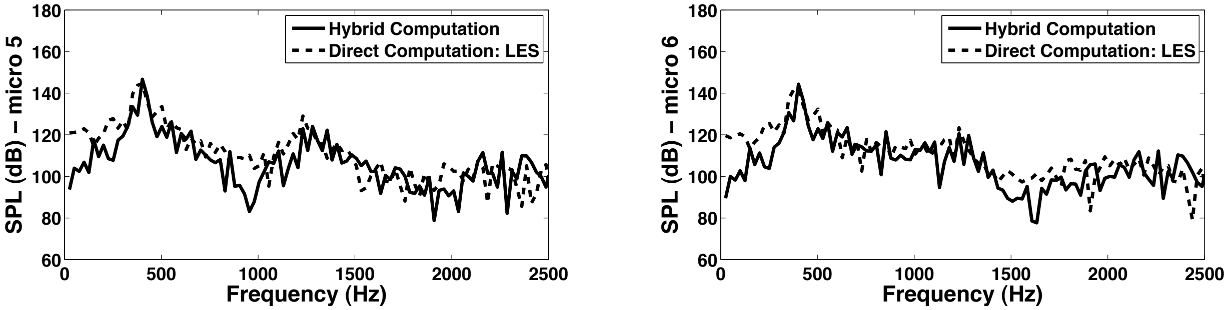


Figure 5. Sound Pressure Levels from the direct and hybrid approaches

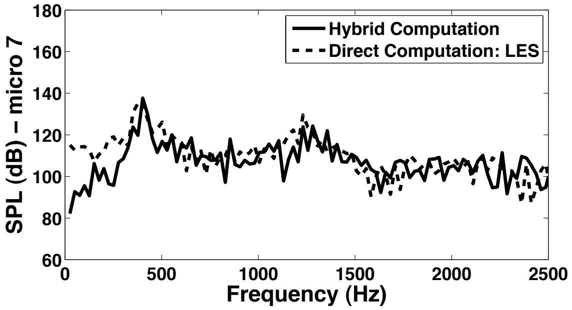


Figure 6. Sound Pressure Levels from the direct and hybrid approaches

Observing with attention fig. **??**, it is noticeable that there are some zones of the spectrum in which an important gap is present between hybrid and direct computations. It is probable, considering the direct computation, that the fluctuations of pressure coming from LES are composed by both acoustic and hydrodynamic contributions. On the other hand, presure fluctuations coming from the hybrid computation are totally due to acoustics. A suitable comparison is then not carried out, and it becomes important to be able to extract acoustics from LES computations in order to evaluate in a proper way the results from the hybrid computation.

V. Filtering a LES pressure field to find the corresponding acoustic field

Velocity fluctuations obtained by LES are composed by both hydrodynamics and acoustics

$$u'_{i,LES} = u'_{i,hyd} + u'_{i,ac} \tag{5}$$

Appling the operator $\partial/\partial t$ to Eq. 5 leads to

$$\frac{\partial u'_{i,LES}}{\partial t} = \frac{\partial u'_{i,hyd}}{\partial t} + \frac{\partial u'_{i,ac}}{\partial t} \tag{6}$$

From linear acoustics, the momentum equation is given by

$$\rho_0 \frac{\partial u'_{i,ac}}{\partial t} = -\frac{\partial p'_{ac}}{\partial x_i} \tag{7}$$

where $[]_0$ and $[]'$ represent respectively the mean and fluctuating flow. Combining this term of eq. 7 into Eq. 6

$$-\frac{1}{\rho_0}\frac{\partial p'_{ac}}{\partial x_i} + \frac{\partial u'_{i,hyd}}{\partial t} = \frac{\partial u'_{i,LES}}{\partial t} \tag{8}$$

Finally the divergence operator to this equation is applied

$$-\frac{\partial}{\partial x_i}\left(\frac{1}{\rho_0}\frac{\partial p'_{ac}}{\partial x_i}\right) + \frac{\partial}{\partial x_i}\left(\frac{\partial u'_{i,hyd}}{\partial t}\right) = \frac{\partial^2 u'_{i,LES}}{\partial x_i \partial t} \tag{9}$$

A. Finding $\frac{\partial \mathbf{u_{i,hyd}}}{\partial \mathbf{x_i}}$

Neglecting viscosity, species diffusion and heat conduction the Navier-Stokes equations for reacting flows read

$$\frac{\partial \rho}{\partial t} + \rho\frac{\partial u_j}{\partial x_j} + u_j\frac{\partial \rho}{\partial x_j} = 0 \tag{10}$$

$$\rho\frac{\partial u_i}{\partial t} + \rho u_j\frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} \tag{11}$$

$$\rho c_p\frac{\partial T}{\partial t} + \rho c_p u_j\frac{\partial T}{\partial x_j} = \dot{q} \tag{12}$$

In the low-Mach number approximation, the thermodynamic pressure P_0 only depends on temperature. The equation of state is simply

$$K_0 = \rho T \tag{13}$$

Replacing Eq. 13 in the left hand side of the Eq. 12 leads to

$$\rho c_p\frac{\partial K_0/\rho}{\partial t} + \rho c_p u_j\frac{\partial K_0/\rho}{\partial x_j} = \dot{q} \tag{14}$$

The temporal derivative of the density in Eq. 10 is combined with Eq. 14. After some algebra, the divergence of the velocity can therefore be expressed as:

$$\frac{\partial u_j}{\partial x_j} = \frac{1}{c_p K_0}\dot{q} \tag{15}$$

This velocity field is supposed to be composed only by hydrodynamics, due to the fact that in the low Mach number model the acoustic wave length is infinitely long. One can state that the divergence of the fluctuating velocity is

$$\frac{\partial u'_{j,hyd}}{\partial x_j} = \frac{1}{c_p K_0}\dot{q}' \tag{16}$$

B. Finding the Acoustic Pressure

Injecting Eq. 16 into Eq. 9 leads to

$$-\frac{\partial}{\partial x_i}\left(\frac{1}{\rho_0}\frac{\partial p'_{ac}}{\partial x_i}\right) = \frac{\partial}{\partial t}\left(\frac{\partial u'_{i,LES}}{\partial x_i} - \frac{\dot{q}'}{c_p K_0}\right) \tag{17}$$

or in the frequency domain

$$\frac{\partial}{\partial x_i}\left(\frac{1}{\rho_0}\frac{\partial \hat{p}_{ac}}{\partial x_i}\right) = i\omega\frac{\partial \hat{u}_{i,LES}}{\partial x_i} - i\omega\frac{\hat{\dot{q}}}{c_p K_0} \tag{18}$$

Finally, multiplying everywhere by γP_0

$$\frac{\partial}{\partial x_i}\left(c^2\frac{\partial \hat{p}_{ac}}{\partial x_i}\right) = \underbrace{i\omega\gamma P_0\frac{\partial \hat{u}_{i,LES}}{\partial x_i}}_{T1} - \underbrace{i\omega\frac{\gamma P_0\hat{\dot{q}}}{c_p K_0}}_{T2} \tag{19}$$

It has been found by the author that the contribution of $T2$ can be neglected. As a consequence, one can state that for a reactive flow the divergence of the hydrodynamic velocity field ($\frac{\partial u_{hyd}}{\partial x_i}$) can be considered zero as typically done for non-reactive flows. Equation 19 simplifies

$$\mathcal{F}(\hat{p}_{ac}) = \frac{\partial}{\partial x_i}\left(c^2\frac{\partial \hat{p}_{ac}}{\partial x_i}\right) = i\omega\gamma P_0\frac{\partial \hat{u}_{i,LES}}{\partial x_i} \tag{20}$$

VI. LES Vs Hybrid Results

Figures 7 and 8 shows the Sound Pressure Level (SPL) given by the solution of Eq. 20 (LES 'Filtered') and the hybrid computation for microphones 5, 6 and 7.

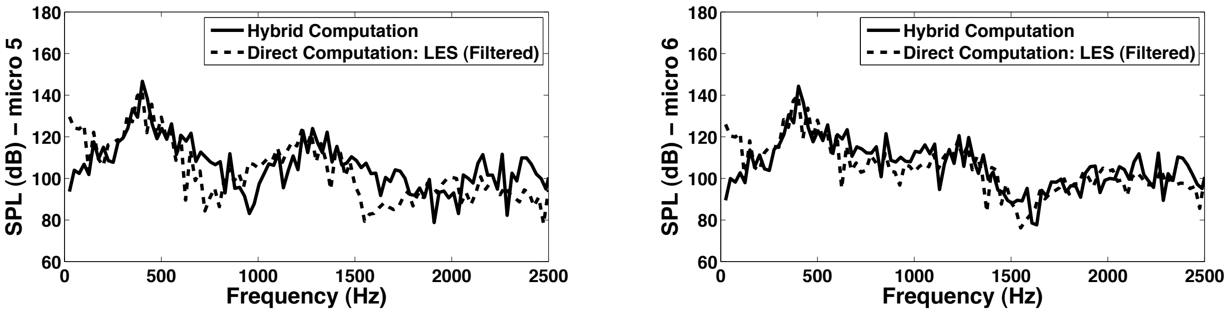


Figure 7. Sound Pressure Levels from the direct and hybrid approaches

After extracting the acoustic field from the complete pressure fluctuation field, it is seen that results match pretty well with the values predicted by the hybrid method, especially for microphones 6 and 7. There is a high contain of hydrodynamic fluctuations at low frequencies (before 400 Hz) that has been removed. Also

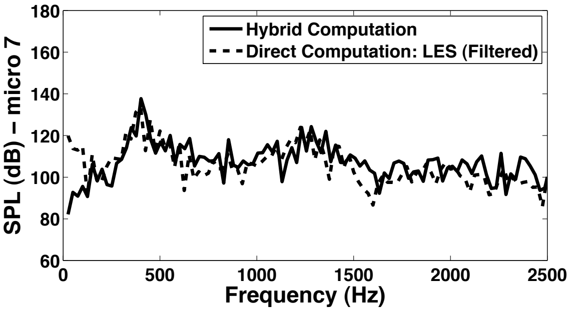


Figure 8. Sound Pressure Levels from the direct and hybrid approaches

the local minimum around 1500 Hz shown in the SPL spectrum for microphone 6 is well recovered after extracting acoustics from LES. The acoustic signal corresponding for microphone 5 present on the contrary, important differences with respect to the one computed by the hybrid approach. The local minima around 1000 Hz corresponding to the hybrid method is recover at a lower frequency (around 750 Hz).

Note that some caution must be taken when computing the Fourier transform of the divergence of the velocity (See T1 Eq. 19). If a rectangular window is applied to the temporal signal, a really good correspondance is seen only for frequencies lower than 400 Hz. On the other hand, if a gaussian window is applied, a good filtering is seen over the entire frequency band, except at a very low frequencies. This is the case for the spectra shown (Figs. 7 and 8) where the signal does not capture the good acoustic level before 200 Hz.

VII. Conclusions

It has been shown that when aiming to compute combustion noise in confined domains by a direct method (LES in this case), the pressure field obtained contains both acoustics and hydrodynamics. Moreover, hydrodynamics was proved to be strong at low frequencies and hence, using a LES pressure field to estimate combustion noise is not enough: extracting acoustics from the complete pressure field is essential. In this paper, a procedure to separate acoustic from hydrodynamics has been proposed. A validation of this procedure is made by comparing the resulting signal to combustion noise predicted by a Hybrid method.

Acknowledgments

This work was supported by the Fondation de Recherche pour l'aéronautique et l'espace (FRAE) through the BRUCO project. The authors also gratefully acknowledge the Centre Informatique National de l'Enseignement Supérior (CINES) for giving access to parallel computers and École Centrale Paris for the interesting discussions and experimental data.

References

[1] A. Oberai, F. Roknaldin and T. Hughes "Computation of Trailing-Edge Noise Due to Turbulent Flow over an Airfoil" AIAA Journal, **40** 2206 - 2216 (2002).

[2] D.J. Bodony and S. K. Lele "On the current status of jet noise predictions using Large-Eddy Simulation" AIAA Journal, **46** 364 - 380 (2008).

[3] H. Pitsch and H. Steiner, "Large-eddy simulation of a turbulent piloted methane/air diffusionn flame (Sandia flame D)" Phys. Fluids , **12**, 2541-2554 (2000).

[4] M. Ihme, H. Pitsch and H. Bodony "Radiation of Noise in Turbulent flames" Proc. Comb. Inst., **32**, 1545-1554 (2009).

[5] F. di Mare, W. Jones and K. Menzies "Large eddy simulation of a model gas turbine combustor" Combust. Flame, **137**, 278-294 (2004).

[6] A. S. Lyrintzis "Integral acoustic methods: From the (cfd) near-field to the (acoustic) far-field" Int. J. Aeroacoustics, **2**, 95-128 (2003).

[7] C. Bailly, C. Bogey and S. Candel "Modelling of sound generation by turbulent reacting flows" Submitted to Int. J. Aeroacoustics (2009).

[8] M. Wang, S. Moreau G. Iaccarisno and M. Roger "LES prediction of wall-pressure fluctuations and noise of a low-speed

airfoil" Int. J. Aeroacoustics, **3** (2009).

[9] S. Bragg "Combustion noise" J. Inst. of Fuel, **36** 12-16 (1963).

[10] M. J. Lighthill "On sound generated aerodynamically: I. General theory" Proc. R. Soc. London Ser. A, **222** 1 - 32 (1954).

[11] W. C. Strahle "On Combustion Generated Noise" J. Fluid. Mech., **49** 399 - 414 (1971).

[12] S. Kotake "On combustion noise related to chemical reactions" J. Sound Vib., **42** 399-410 (1975).

[13] R. Ewert and W. Schröder. "Acoustic perturbation equations based on flow decomposition via source filtering" J. Comput. Physics, **188** 365 - 398 (2003).

[14] C. Bailly and D. Juvand "Numerical solution of acoustic propagation problems using Linearized Euler Equations" AIAA Journal, **38** 22 - 29 (2000).

[15] M. Roger "Aeroacoustics of wall-bounded flows" Von Karman Institute For Fluid Dynamics. Lecture Series. March 9-13 (2009).

[16] N. Curle "The influence of solid boundaries upon aerodynamic sound" Proc. R. Soc. London Ser. A 231, (1955).

[17] H.G. Davies, J.E. Ffowcs Williams "Aerodynamic sound generation in a pipe" J. Sound Vib., **161** 2 (1981).

[18] C. Schram "A boundary element extension of Curle's analogy for non-compact geometries at low-Mach numbers" J. Sound Vib., **322** 264-281 (2009).

[19] C.F. Silva, M. Leyko, F. Nicoud and S. Moreau "Assesment of combustion noise in a premixed swirled combustor via Large-eddy simulation" Submitted to J. Computer and Fluids, (2009).

[20] O. M. Phillips "On the generation of sound by supersonic turbulent shear layers" J. Fluid Mech., **9** 1-28 (1960).

[21] F. Williams "Combustion Theory" The Benjamin/Cummings Publishing Company (1985)

[22] H. Hassan "Scaling of combustion-generated noise" J. Fluid Mech., **49** 445-453 (1974).

[23] A. Pierce "Acoustics. An introduction to its physical principles and applications" Chapter 1 (1991).

[24] F. Nicoud et al. "Acoustic Modes in Combustors with Complex Impedances and Multidimensional Active Flame" AIAA Journal, (2007).

[25] V. Frayssé et al. "A Set of GMRES Routines fo Real and Complex Arithmetics on High Performance Computers." CERFACS Technical Report, (2003).

[26] A. Lamraoui et al. "Acoustic reconstruction of the fuel and air feeding line impedances in a swirled burner during combustion instabilites" In *3rd European Conference for Aerospace Sciences EUCASS* (2009).

[27] A. Lamraoui et al. "Methodology for on the fly acoustic characterization of the feeding lines impedances in a turbulent swirled combustor" In *Proceedings of ASME Turbo Expo 2010: Power for Land, Sea and Air* (2010).

[28] AVBP Code: *http://www.cerfacs.fr/4-26334-The-AVBP-code.php* and *http://www.cerfacs.fr/4-25719-Publications.php*, (2010)

[29] O. Colin et al "A thickened flame model for large eddy simulations of turbulent premixed combustion" Phys. Fluids **12**, 1843-1863 (2000)

[30] L. Selle et al. "Compressible Large-Eddy Simulation of turbulent combustion in complex geometry on unstructured meshes" Combust. Flame. **137**, 4, 489-505 (2004)

[31] S. Roux et al. "Studies of mean and unsteady flow in a swirled combustor using experiments, acoustic analysis and Large Eddy Simulations" Combust. Flame. **141**, 40-54 (2005)

[32] S. Mendez and F. Nicoud "Large eddy simulation of a bi-periodic turbulent flow with effusion" J. Fluid Mech., **46**, 2623-2633 (2008)

[33] N. Tran "Influence de la condition limite acoustique amont sur les instabilités de combustion de grande amplitude: Conception d'un systéme robuste de contrôle d'impédance" Phd Thesis. Ecole centrale Paris. (2009)

[34] P. Clavin, E. D. Siggia "Turbulent premixed flames and sound generation" Combust. Sci. Technol. **78** 147-155 (1991)

[35] T. Schuller, D.Durox and S.Candel "Dynamics of and Noise Radiated by a Perturbed Impinging Premixed Jet Flame" Combust. Flame. **128** 88-110 (2002)

[36] T.D. Butler, P.J. O'Rourke "A numerical method for two-dimensional unsteady reacting flows" 16th Symp. (Int.) on Combustion. The Co