Hopf algebras and Lie algebras of rooted trees and their quantification

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A. Connes and D. Kreimer introduced a Hopf algebra of rooted trees H_R in order to study a problem of Renormalization in Quantum Field Theories ([1, 8, 9]). This Hopf algebra is commutative, not cocommutative. Its dual is the enveloping algebra of the Lie algebra L^1 of rooted trees. Moreover, D. Kreimer introduced in [2] a Lie algebra L which acts by derivations on H_R. This Lie algebra contains two copies of L^1. We study this Lie algebra L ; in particular, we show that L is a simple infinite-dimensional Lie algebra ([7]).

We introduce a quantum equivalent of the enveloping algebra of L. For this, we need a non-commutative version of H_R, namely the Hopf algebra of planar rooted trees H_{PR} ([3, 4, 5]). We show that H_{PR} is a self-dual Hopf algebra. We then introduce a braided version $(H_{PR})_q$ of H_{PR}, which gives a quantification of H_{PR} ([6]). This braided Hopf algebra is also self-dual ; after a smash product with a torus, we can take the Drinfel'd double $D((H_{PR})_q)$ of $(H_{PR})_q$: this Hopf algebra plays the role of a quantum version of $\mathcal{U}(L)$. We also show that we can obtain the usual quantum groups as subquotient of $D((H_{PR})_q)$.

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